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|  | **Eastern Goldfields College**  ***Year 12 MATHEMATICS METHODS***  ***TEST 1 2019*** |

Name : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**CALCULATOR FREE**

**Total Marks: 28**

**Reading: 2 minutes Time Allowed: 30 minutes**

**Question 1 (5 marks)**

A function defined by has stationary points at and .

(a) Use the second derivative to show that one of the stationary points is a local maximum and the other a local minimum. (3 marks)

(b) Determine the coordinates of the point of inflection of the graph of . (2 marks)

**Question 2 (4 marks)**

If the radius of a sphere is measured with an error of at most 4%, estimate the percentage error in the volume of the sphere.

**Question 3 (12 marks)**

Determine the gradient function ****for each of the following.

Leave your answers with positive indices, where necessary. Do not simplify.

(a)  (Where *a* is a positive constant) (1 mark)

(b)  (2 marks)

(c)  (3 marks)

(d) y = [3 + cos(x/2)]4 (3 marks)

(e)  (3 marks)

**Question 4 (7 marks)**



(a) Evaluate:

(i) *f* (2) (1 mark)

(ii) *f*  ′(2) (1 mark)

(iii) *f*  ′′(2) (1 mark)

(b) (i) Sketch the graph of *f* (*x*) over the domain  on the axes provided. (1 mark)



(ii) With reference to your sketch, explain the significance of each answer

from part (a). (3 marks)

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**CALCULATOR ASSUMED**

**Total Marks: 27**

**Reading: 2 minutes Time Allowed: 27 minutes**

**Question 5 (4 marks)**

A spherical balloon is being inflated by pumping gas into it at a rate of 5 m3/minute.

Determine the rate at which the diameter is increasing when the radius is 1 m.

**Question 6 (4 marks)**

A population, *y*, increases according to the differential equation:



The population at the start of 2000 has size 1 000.

(a) State the equation for population, *y*, in terms of *t*. (1 mark)

(b) State the population size when *t* = 5. (1 mark)

(c) Determine the doubling time for the population. (2 marks)

**Question 7 (11 marks)**

The function has a global minimum value over the domain .

(a) Complete the following table. (2 marks)

|  |  |
| --- | --- |
| *x* |  |
| 0.5 |  |
| 0.6 |  |
| 0.7 |  |
| 0.8 |  |

(b) Explain why the table shows that a local minimum exists in the domain. (1 mark)

(c) Sketch the graph of *y* = *h* (*x*) over the domain given on the axes provided, labelling

clearly any important points. (2 marks)



(d) State the minimum value of *h* (*x*), correct to three decimal places, over the

stated domain. (1 mark)

***Question 7 continued***

(e) Use your calculator to determine the derivative *h* ′(*x*), giving your answer in fractional

form, and with positive indices. (1 mark)

(f) Explain how to use the derivative *h* ′(*x*) and your calculator to determine any stationary

point of *h* (*x*)*.* State the *x* value of any stationary point. (2 marks)

(g) Use the “Sign Test”, by completing the following table, to prove that the stationary

point found in (f) is a local minimum. (1 mark)

|  |  |  |  |
| --- | --- | --- | --- |
| *x* |  |  |  |
| *h* ′(*x*) |  |  |  |

(h) State the nature of concavity of *h* (*x*). (1 mark)

Question 8 (8 marks)

A storage container of volume cm3 is to be made in the form of a right circular cylinder with one end open. The material for the circular end costs 12c per square centimetre and for the curved side costs 9c per square centimetre.

a) Show that the cost of materials for the container is cents, where is the radius of the cylinder. (4 marks)

b) Use calculus techniques to determine the dimensions of the container that minimise its material costs and state this minimum cost. (4 marks)